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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2016

SECOND YEAR [BATCH 2015-18]

Date : 21/12/2016 **STATISTICS [General]**

Time : 11 am – 1 pm **Paper : III** Full Marks : 50

[Use a separate Answer Book for each Group]

Group – A

***Answer <u>any two</u> questions*:** [2×5]

1. If a random sample of size *n* is selected from the finite population which consists of the integers 1,2, …, *N*, show that $V(\bar{X}) = \frac{(N+1)(N-n)}{12n}$, where X_i is the number selected in the i^{th} draw and $\bar{X}$ is the mean. [Assume SRSWOR and you can also use the standard results of SRSWOR] [5]

2. If $X_1, X_2, \cdots, X_n$ constitute a random sample from a normal population with μ = 0, show that $\sum_{i=1}^{n} X_i^2 \big/ n$ is an unbiased estimator of σ^2. [σ^2 is the variance of the normal population]. [5]

3. If $V_1, V_2, \cdots, V_n$ and $W_1, W_2, \cdots, W_n$ are independent random samples of size *n* from normal populations with the means $\mu_1 = (\alpha + \beta)$ & $\mu_2 = (\alpha - \beta)$ respectively and common variance $\sigma^2 = 1$ i.e. $V_i \sim N(\alpha+\beta, 1)$ & $W_i \sim N(\alpha-\beta, 1)$ $\forall\, i = 1, 2, \cdots, n$. Find the maximum likelihood estimators for α & β [You can use the standard MLEs for the normal distribution parameters, without deriving these]. [5]

4. Explain what do you mean by the power of a test? Define the probabilities of type I & type II errors. Which error is more severe? Can we minimize both? If yes how and if not why? [5]

***Answer <u>any two</u> questions*:** [2×10]

5. a) Consider a $N(\mu, \sigma^2)$ population where μ & σ^2 are unknown. We are to test the null hypothesis $H_0 : \mu = \mu_0$ against the alternatives $H_{1A} : \mu > \mu_0$; $H_{1B} : \mu < \mu_0$; $H_{1C} : \mu \neq \mu_0$ on the basis of the random sample $(X_1, X_2, \cdots, X_n)$ of size *n*, construct critical regions (of size α) corresponding to the three alternatives. [5]

   b) A random sample $X_1, X_2, \cdots, X_n$ is drawn from an infinite population with unknown mean μ & unknown variance σ^2. Show that the sample variance $S^2 = \frac{1}{n}\sum (X_i - \bar{X})^2$ is not an unbiased estimator of σ^2. Find the unbiased estimator of σ^2. [5]

6. In statistical inference, explain the application of Pearsonian statistics for testing – [10]
   - Goodness of fit
   - Independence of two attributes

7. (i) Given $P\left[F_{10,12} > 2.753\right] = 0.05 = P\left[F_{1,12} > 4.747\right]$. Find $P\left[F_{12,10} > \frac{1}{2.753}\right]$ and $P\left[-\sqrt{4.747} > t_{12} < \sqrt{4.747}\right]$.

[The letters *F* and *t* denote respectively *F*-distribution & *t*-distribution. The integers affixed to *F* & *t* are the relevant degrees of freedom]. [5]

(ii) Let X_1 and X_2 constitute a random sample from a normal population with $\sigma^2 = 1$. If the null hypothesis $\mu = \mu_0$ is to be rejected in favour of the alternative hypothesis $\mu = \mu_1 > \mu_0$ when $\bar{X} > \mu_0$, what is the size of the critical region? [5]

8. Random samples of sizes n_1 & n_2 are drawn from two independent normal populations $N(\mu_1, \sigma_1^2)$ & $N(\mu_2, \sigma_2^2)$. Explain how do you come up with the 100 (1–α)% confidence interval for $(\mu_1 - \mu_2)$, considering the two cases when (i) σ_1^2 & σ_2^2 are known (ii) σ_1^2 & σ_2^2 are unknown. [4+6]

Group – B

***Answer <u>any two</u> questions*:** [2×5]

9. Briefly explain the steps to construct price index number of two different time periods. [5]

10. Explain uses of index number. Clarify the concept 'Purchasing power of money'. [5]

11. Distinguish between seasonal variation and cyclical variations. [5]

12. Discuss the moving average method for determining trend in a time series. What are its merits and demerits? [5]

***Answer <u>any one</u> question*:** [1×10]

13. Find the trend value for the year 1996 by fitting a second degree polynomial to the following data: [10]

Year:	1993	1994	1995	1996	1997
Sales (,000 Rs)	16	18	19	20	24

14. What do you mean by C.L.I.N? Describe the different steps in construction of CLIN for jute workers of West Bengal. [10]

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